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FIRST OPPORTUNITY EXAMINATION QUESTION PAPER

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INSTRUCTIONS:

(i) This Question Paper consists of **four** pages (including this front page).

(ii) Answer **all** the questions in Section A and **any three questions** in Section B.

(iii) Only **non-programmable calculators** may be used.

(iv) Number the questions clearly and present your solutions using proper mathematical terminology.

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EXAMINATIONS

SECTION A

Question 1.

1.1 Find the values of s and k for which the vector

$$\mathbf{v} = \begin{pmatrix} 2 \\ s \\ 4 \end{pmatrix}$$

is **not** a linear combination of the vectors

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \quad \mathbf{u}_3 = \begin{pmatrix} 1 \\ 1 \\ k \end{pmatrix}.$$

[6]

Question 2.

2.1 Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 4 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}.$$

(i) For what value(s) of r is $D = BC$ invertible? [4]

(ii) Compute $(AB)C$. [4]

(iii) Find the determinant of $(AB)C$. [2]

(iv) Find $\left((AB)^{-1}C\right)^T$. [4]

2.2 Let $X = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$, $Y = \begin{pmatrix} 0 \\ -3 \\ -1 \end{pmatrix}$ and $Z = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$ be vectors in $\mathbb{R}^3$.

(a) Find the vector equation of the line L through the points X and Z. [4]

(b) Find the equation of the hyerplane H through X, Y and Z. [4]

(c) Determine whether the line L and the hyperplane H are parallel, orthogonal or neither. [3]

Question 3.

3.1 Let $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \middle| x + 2y - z + w = 0 \right\}$ and $u_1, u_2 \in V$, where $u_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$ and $u_2 = \begin{pmatrix} -2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

Extend $\{u_1, u_2\}$ to a basis of V. [9]

[40]

SECTION B

Question 4.

4.1 Let $S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| -x \leq y - z \right\}$ and $T = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| 2x + y = 0 \text{ and } x + 4z = 0 \right\}$ be sets in $V = \mathbb{R}^3$.

(a) Determine whether S is a subspace of V or not. Explain! [5]

(b) Find a basis and the dimension of a subspace T. [4]

4.2 Determine the rank and nullity of the matrix $A = \begin{pmatrix} 0 & 0 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 & -18 \\ 1 & 2 & 2 & -1 & -11 \\ -1 & -2 & -2 & 1 & 11 \end{pmatrix}$. [5]

4.3 Assume $u_1, \ldots, u_k$ are linearly independent. Prove that every vector $v \in \text{span}\{u_1, \ldots, u_k\}$ can be written as a linear combination of $u_1, \ldots, u_k$ in a unique way. [6]

[20]

Question 5.

5.1 Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ and $S : \mathbb{R}^2 \to \mathbb{R}^2$ be defined as follows: $T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} -x - y \\ -x - y \end{pmatrix}$, S is the unique linear transformation satisfying $S(e_1 - 4e_2) = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$ and $S(2e_1 - 3e_2) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(a) Compute the image and the kernel of T. [6]

(b) Find $S(v)$, $v \in \mathbb{R}^2$. [5]

(c) Find $S^{-1}(v)$. [4]

5.2 Solve the following system using Gaussian elimination and comment on the solution type.

$$\begin{cases} y + 2z = 2 \\ 2x + y - 2z = 3 \\ 4x - y - 10z = 4 \end{cases}$$

[5]

[20]

Question 6.

6.1 Extend $\{u_1, u_2\}$ to a basis of $\mathbb{R}^4$, where

$$u_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 2 \end{pmatrix} \text{ and } u_2 = \begin{pmatrix} 1 \\ 2 \\ -2 \\ 4 \end{pmatrix}.$$

[7]

6.2 Let $T : V \to V$ be a linear map. Prove that if $\ker(T) = \{\mathbf{0}\}$, then T is injective. [6]

6.3 Let $V = \mathbb{R}^3$ and $\mathcal{B} = (e_2, e_1 - e_3, -2e_1), \mathcal{B}' = (e_1, e_3 - e_1, e_2)$ be bases of V. Compute ${}_{\mathcal{B}}\mathcal{M}_{\mathcal{B}'}(\mathrm{id}_v)$. [7]

[20]

Question 7.

7.1 Find the value(s) of t for which the vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$$

form a basis for $\mathbb{R}^3$. [5]

7.2 Let $V = \mathbb{R}^2$, and define

$$U_1 = \left\{ \begin{pmatrix} x \\ 0 \end{pmatrix} \mid x \in \mathbb{R} \right\}, \qquad U_2 = \left\{ \begin{pmatrix} 0 \\ y \end{pmatrix} \mid y \in \mathbb{R} \right\}.$$

(i) Find the spanning set of $U_1 \cap U_2$. [2]

(ii) Is $U_1 \oplus U_2 = V$? Give a clear reason. [4]

7.3 Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ -1 & 2 & 1 & -1 \\ 2 & 3 & 4 & 1 \end{pmatrix}.$$

Find a basis for the image (column space) of A. [9]

[20]