UNAM

FACULTY	AGRICULTURE, ENGINEERING & NATURAL SCIENCES		
DEPARTMENT	COMPUTING, MATHEMATICAL & STATISTICAL SCIENCES		
SUBJECT	CALCULUS I		
SUBJECT CODE	S3511MC		
DATE	MAY/JUNE 2026		
DURATION	**3 HOURS**	**MARKS**	100

FIRST OPPORTUNITY EXAMINATION

Examiner: Dr J. Mushanyu (UNAM)

Moderator: Dr P. Haihambo (UNAM)

This question paper consists of **five** pages (including this front page).

Instructions

- **Candidates must attempt ALL questions in Section A and any THREE questions in Section B.**
- **Each question should start on a fresh page.**
- **Answers must be correctly numbered.**
- **Only *non-programmable calculators* may be used.**

UNIVERSITY OF NAMIBIA EXAMINATIONS

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S3511MC

CALCULUS I

May/June 2026
Time : 3 hours

Candidates must attempt ALL questions in Section A and any THREE questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates must attempt ALL questions being careful to number them A1 to A4.

A1. (a) State whether each of the following statements is true or false.

(i) The sequence $a_n = \dfrac{n}{n+1}$ is bounded above by 1. [1]

(ii) If $\lim\limits_{n\to\infty} a_n = L$, then $\lim\limits_{n\to\infty}(3a_n - 2) = 3L - 2$. [1]

(iii) The function $f(x) = |x|$ is differentiable at $x = 0$. [1]

(iv) If f and g are functions such that $f(x) = g(x)$ for all x in their respective domains, then their graphs are identical. [1]

(b) Determine the general term of the sequence $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dfrac{5}{6}, \cdots$ [2]

(c) Given that $\lim\limits_{n\to\infty}(1 + x^n)^{\frac{1}{n}} = 1$ if $|x| < 1$, determine $\lim\limits_{n\to\infty}\left(\dfrac{2^n + 4^n}{5^n + 6^n}\right)^{\frac{1}{n}}$. [4]

A2. (a) Evaluate the following limits:

(i) $\lim\limits_{x\to 2}\dfrac{x^2 - 4}{x - 2}$ (ii) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4} - 2}{x}$. [2,3]

(b) Consider the function

$$f(x) = \begin{cases} ax + 1, & x < 1, \\ 3, & x = 1, \\ 2x + b, & x > 1. \end{cases}$$

Determine the values of a and b such that f is continuous at $x = 1$. [5]

A3. (a) (i) Give the $\epsilon - \delta$ definition of a limit of a function $f(x)$ at an accumulation point $x = x_0$ of the domain of f. [2]

(ii) Using the definition in (a)(i), prove that $\lim_{x \to 1} \dfrac{9 - x^2}{1 + x} = 4$. [4]

(b) Let $f(x) = \sqrt{5 - x}$ and $g(x) = \sqrt{x + 4}$. Find $(f \circ g)(x)$ and state its domain. [4]

A4. Let $f(x) = \ln(x^3 - 3x^2 + 2x)$.

(a) Find $f'(x)$. [4]

(b) Determine the equation of the tangent line to the curve $y = f(x)$ at the point where $x = 3$. [4]

(c) Find the x-values of the stationary points of $f(x)$. [2]

SECTION B (60 marks)

Candidates must attempt any THREE questions being careful to number them B5 to B8. Candidates are advised that it is in their own interest to concentrate on obtaining complete solutions to questions rather than producing fragmentary answers to a larger number of questions.

B5. (a) (i) Define precisely what is meant by saying that $\lim_{n\to\infty} u_n = \ell$, where $\ell \in \mathbb{R}$. [2]

(ii) Using the definition of a limit of a sequence, show that $\lim_{n\to\infty} a^n = 0$ where $0 < |a| < 1$. [5]

(b) Evaluate $\lim_{n\to\infty} n\left(\sqrt{n^2+8} - \sqrt{n^2+3}\right)$. [4]

(c) Find the limit of the sequence $\left\{\sqrt{2},\ \sqrt{2\sqrt{2}},\ \sqrt{2\sqrt{2\sqrt{2}}},\ \ldots\right\}$. [4]

(d) A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$. Show by induction that $\{a_n\}$ is bounded above by 3 for all $n \in \mathbb{N}$. [5]

B6. (a) Let

$$f(x) = \begin{cases} 1, & x < 0, \\ x, & 0 < x < 1, \\ 2 - x, & 1 < x < 3, \\ x - 4, & x > 3. \end{cases}$$

(i) Is it possible to define f at $x = 0$ in such a way that f becomes continuous at $x = 0$? If so, state the value of $f(0)$. [2]

(ii) Is it possible to define f at $x = 1$ in such a way that f becomes continuous at $x = 1$? If so, state the value of $f(1)$. [2]

(iii) Is it possible to define f at $x = 3$ in such a way that f becomes continuous at $x = 3$? If so, state the value of $f(3)$. [2]

(b) Show that if $ad - bc \neq 0$, then the function $f(x) = \dfrac{ax + b}{cx + d}$ is one-to-one. [4]

(c) Find the inverse function of $f(x) = \ln(x + \sqrt{x^2 + 1})$. [4]

(d) The interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+1)^{2n}}{(n+1)^2\, 5^n}$ is $[a - \sqrt{b},\ a + \sqrt{b}]$. Determine the values of a and b. [6]

B7. (a) Evaluate the following limits:

(i) $\lim_{x\to 0} \dfrac{e^{2x}-1-2x}{x^2}$, (ii) $\lim_{x\to\infty} \dfrac{\ln x}{x}$. [3,3]

(b) Use differentiation techniques to find $\dfrac{dy}{dx}$ for each of the following

(i) $y = x^x$, where $x > 0$, (ii) $y = \sin(\ln(x^3 - 4x))$. [4,4]

(c) If $y = \left(\sin^{-1} x\right)^2 + 2\sin^{-1} x$. Determine $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 2$. [6]

B8. (a) (i) State the Intermediate Value Theorem. [2]

(ii) Use the Intermediate Value Theorem to show that the equation

$$x^3 + x - 1 = 0$$

has at least one root in the interval $(0, 1)$. [2]

(b) (i) State Rolle's Theorem. [2]

(ii) Use Rolle's Theorem to show that the equation

$$x^3 + 3x + 1 = 0$$

has at most one real root. [2]

(c) (i) State the Mean Value Theorem. [2]

(ii) Use the Mean Value Theorem to show that if $0 < a < b$, then

$$\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}.$$

[4]

(d) (i) State without proof Leibniz's Theorem for the nth derivative of a product uv. [2]

(ii) If

$$y = x^2 e^x,$$

use Leibniz's Theorem to find $y^{(n)}$, where $n \geq 2$. [4]

END OF QUESTION PAPER