UNAM

UNIVERSITY OF NAMIBIA

FACULTY	AGRICULTURE, ENGINEERING AND NATURAL SCIENCES
DEPARTMENT	ELECTRICAL AND COMPUTER ENGINEERING
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SUBJECT CODE	TCEE 3831
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DURATION	3 HOURS / **MARKS** 100

First Opportunity Examination

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This Question paper consists of …**3**…. Pages excluding this front page.

Instructions

1. Answer **ALL FOUR** questions
2. Marks for different sections are shown on the right hand margin
3. Write your student number on each page
4. Start each question on a separate page
5. Clearly show your reasoning
6. Present all your answers to three significant figures.
7. Additional materials: Z-transform Table, DFT Table.

Question 1 [25 Marks]

1.1. (a) What is digital signal processing and how is it applied to 1D, 2D, and 3D signal processing applications ? **[5 Marks]**

(b) Comment about the linearity, stability, time-invariance and causality for the following DSP system:

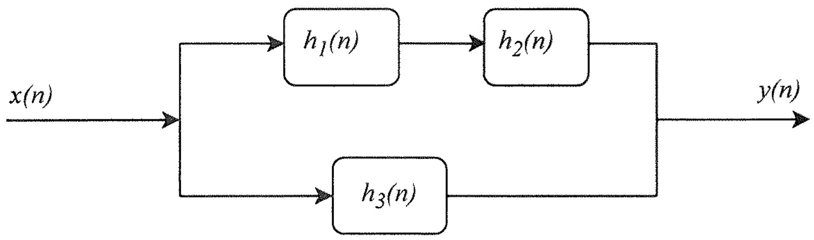
$$y(n) = 2x(n+1) + [x(n-1)]^2$$

[8 Marks]

1.2. (a) Explain the concepts of convolution and correlation of two signals and How can correlation be computed using convolution? **[4 Marks]**

(b) Three discrete time LTI systems are connected as shown in figure. 1.2.1. Determine the overall unit sample response of the connection if,

$$h_1(n) = (\tfrac{1}{2})^n u(n), \qquad h_2(n) = (\tfrac{1}{4})^n u(n), \quad \text{and} \quad h_3(n) = u(n)$$



[8 Marks]

Figure 1.2.1: Connection diagram of three discrete time systems

Question 2 [25 Marks]

2.1. (a) Using properties of Z – transform, determine the $X(z)$ for the following sequences,

(i) $x(n) = u(-n+1)$ **[3 Marks]**

(ii) $x(n) = 2^n u(n-2)$ **[3 Marks]**

(b) Find the inverse Z-transform of

$$H(z) = \frac{z(z^2 - 4z + 5)}{(z-3)(z-1)(z-2)}$$

for ROC $2 < |z| < 3$ using partial fraction method. **[6 Marks]**

2.2. (a) What is the relation between Discrete-Time Fourier Transform, Z-Transform and Discrete Fourier transform? **[5 Marks]**

(b) Perform circular convolution of the following two sequences $x_1(n)$ and $x_2(n)$ by means of DFT & IDFT and compare it with linear convolution.

$$x_1(n) = [1, 2, 0, 1] \quad \text{and} \quad x_2(n) = [2, 2, 1, 1]$$ **[10 Marks]**

Question 3 [25 Marks]

3.1. (a) Calculate the percentage saving in calculations in a 256-point radix-2 FFT, when compared to direct DFT **[4 Marks]**

(b) An 8-point sequence is given by,

$$x(n) = \begin{cases} 2u(n), & 0 \leq n \leq 3 \\ u(n), & 4 \leq n \leq 7 \end{cases}$$

(i) What is the meaning of radix-2 DIT-FFT ? **[2 Marks]**

(ii) Compute 8-point DFT of x(n) using radix-2 DIF-FFT. **[6 Marks]**

3.2. (a) Realize the following system with minimum number of multipliers

$$H(z) = (1 + \frac{1}{3}z^{-1} + z^{-2})(1 + \frac{1}{5}z^{-1} + z^{-2})$$ **[5 Marks]**

(b) LTI system is described by the following difference equation

$$y(n) = -\frac{13}{12}y(n-1) - \frac{9}{24}y(n-2) - \frac{1}{24}y(n-3) + x(n) + 4x(n-1) + 3x(n-2)$$

(i) Identify the filter type based on its transfer function **[2 Marks]**

(ii) Obtain direct form-I and direct form-II structures **[6 Marks]**

Question 4 [25 Marks]

4.1. (a) What are the advantages and disadvantages of FIR filters over IIR filters ? **[4 Marks]**

(b) Design a linear-phase FIR filter using windowing method with the following specifications:

1. $M = 7$ and $\omega = 1$ rad/sample

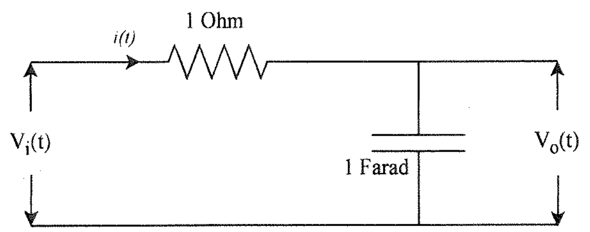
2. Desired frequency response:

$$H(e^{jw}) = \begin{cases} e^{-j3w}, & -\omega_c \leq w \leq \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases}$$

3. The window $w(n)$ has main lobe width is equal to $\frac{8\pi}{M}$ and peak side-lobe magnitude (or relative side-lobe level) is equal to -31dB **[11 Marks]**

4.2. (a) What are the advantages and disadvantages of bilinear transformation ? **[4 Marks]**

(b) Obtain the digital filter equivalent of analog filter shown in Fig. 4.2.1, using bilinear transformation. Assume the sampling frequency $F_s = 8F_c$, where F_c is the cut-off frequency of the filter.



[6 Marks]

Figure 4.2.1: Analog Filter

[Appx-1]

Common z-Transform Pairs

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3	$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4	$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
5	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
6	$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7	$\cos(\omega_0 n)u(n)$	$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
8	$\sin(\omega_0 n)u(n)$	$\frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
9	$a^n\cos(\omega_0 n)u(n)$	$\frac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
10	$a^n\sin(\omega_0 n)u(n)$	$\frac{1-az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

z-Transform Properties

Property	Time Domain	z-Domain	ROC
Notation:	$x(n)$	$X(z)$	ROC: $r_2 < \lvert z\rvert < r_1$
	$x_1(n)$	$X_1(z)$	ROC_1
	$x_2(n)$	$X_2(z)$	ROC_2
Linearity:	$a_1x_1(n)+a_2x_2(n)$	$a_1X_1(z)+a_2X_2(z)$	At least $\text{ROC}_1 \cap \text{ROC}_2$
Time shifting:	$x(n-k)$	$z^{-k}X(z)$	At least ROC, except $z=0$ (if $k>0$) and $z=\infty$ (if $k<0$)
z-Scaling:	$a^n x(n)$	$X(a^{-1}z)$	$\lvert a\rvert r_2 < \lvert z\rvert < \lvert a\rvert r_1$
Time reversal	$x(-n)$	$X(z^{-1})$	$\frac{1}{r_1} < \lvert z\rvert < \frac{1}{r_2}$
Conjugation:	$x^*(n)$	$X^*(z^*)$	ROC
z-Differentiation:	$n\,x(n)$	$-z\frac{dX(z)}{dz}$	$r_2 < \lvert z\rvert < r_1$
Convolution:	$x_1(n) * x_2(n)$	$X_1(z)X_2(z)$	At least $\text{ROC}_1 \cap \text{ROC}_2$

DFT Properties

Property	Time Domain	Frequency Domain
Notation:	$x(n)$	$X(k)$
Periodicity:	$x(n)=x(n+N)$	$X(k)=X(k+N)$
Linearity:	$a_1x_1(n)+a_2x_2(n)$	$a_1X_1(k)+a_2X_2(k)$
Time reversal	$x(N-n)$	$X(N-k)$
Circular time shift:	$x((n-l))_N$	$X(k)e^{-j2\pi kl/N}$
Circular frequency shift:	$x(n)e^{j2\pi ln/N}$	$X((k-l))_N$
Complex conjugate:	$x^*(n)$	$X^*(N-k)$
Circular convolution:	$x_1(n)\otimes x_2(n)$	$X_1(k)X_2(k)$
Multiplication:	$x_1(n)x_2(n)$	$\frac{1}{N}X_1(k)\otimes X_2(k)$
Parseval's theorem:	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$