UNAM
UNIVERSITY OF NAMIBIA

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DEPARTMENT:	ELECTRICAL AND COMPUTER ENGINEERING
SUBJECT:	APPLIED ELECTROMAGNETICS
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DURATION:	3 HOURS **MARKS: 100**

FIRST OPPORTUNITY EXAMINATION

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This exam consists of ten (10) pages excluding this front page.

Instructions

1. Close book examination.
2. Answer **All** questions and clearly show your working and reasoning.
3. Write your student number on each page.
4. Marks for different sections are shown on the right-hand margin.
5. Additional information (formula and tables) are provided.
6. Only scientific calculators are allowed.

Answer all questions.

Part 1: Coulomb's Law, Electric Field Intensity and Gauss's Law

Question 1 .... Total: 25

(a) Coulomb formulated a law based on experiment(s), he conducted for his electrostatic analysis. As a student, assist other students better understand Coulomb's law as applied to point charges. In your assistance use,

(i) Infographics to illustrate the law. [2]
(ii) Mathematical formula and statements. [3]
(iii) Examples to explain how Coulomb's law is employed in charge distribution scenarios other than point charges. [3]

(b) Consider the electric field $\mathbf{E}$ given as

$$\mathbf{E} = \frac{-8x}{y}\mathbf{a}_x + \frac{4x^2}{y2}\mathbf{a}_y$$

(i) Find the equation of the streamline. [4]
(ii) Using your equation in (i) find the particular streamline, passing through the point $P(1, 4, -2)$. [2]

(c) In general, if the charge distribution is known, electric field flux can be calculated using Gauss's law. In this instance, the unknown quantity that needs to be calculated is included inside an integral equation. If we can select a closed surface which satisfies two conditions, than the solution could be easily obtained.

(i) Write down and briefly describe these two neccessary conditions. [2]
(ii) Assume that the conditions in your response in (i) are not satisfied, describe and provide examples of the way you will apply Gauss law to compute electric flux. [4]

(d) Given the electric flux density, $\mathbf{D} = 0.3r^2\mathbf{a}_r \ nC/m^2$ in free space.

(i) find the electric field E at point $P(r = 2, \theta = 25°, \phi = 90°)$. [2]
(ii) find the total charge within the sphere $r = 3$. [3]

Part 2: Energy and Potential & Conductors and Dielectrics

Question 2 .... Total: 25

(a) If we take the zero reference for potential at infinity, find the potential at $(0, 0, 2)$ caused by this charge configuration in free space

(i) a 12 nC/m on the line $\rho = 2.5\ m$, $z = 0$ [3]
(ii) a point charge of 18 nC at $(1, 2, -1)$ [3]

(b) A dipole of moment $\mathbf{p} = 6\mathbf{a}_z\ nC \cdot m$ is located at the origin in free space.
(i) Find the potential V at $P(r = 4, \theta = 20°, \phi = 0°)$. [3]
(ii) Find electric field $\mathbf{E}$ at point P. [3]

(c) The concept of current is logically followed by a discussion of the conservation of charge and the continuity equation.

(i) State the principle of conservation of charge. [2]
(ii) The continuity equation follows from the principle of conservation of charge considering any region bounded by a closed surface. Show that the point form of the continuity equation is given by [4]

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$$

(d) Find the magnitude of the current density in a sample of silver for which the conductivity, $\sigma = 6.17 \times 10^7\ S/m$ and the mobility of an electron, $\mu_e = 0.0056\ m2/V \cdot s$ if

(i) the drift velocity is 1.5 $\mu m/s$; [2]
(ii) the electric field intensity is 1 mV/m; [2]
(iii) the sample is a cube 2.5mm on a side having a voltage of 0.4 mV between opposite faces; [2]
(iv) the sample is a cube 2.5 mm on a side carrying a total current of 0.5 A [1]

Part 3: Magnetostatics

Question 3 .... Total: 25

(a) The rectangular loop shown in Figure 1 consists of 20 closely wrapped turns and is hinged along the z−axis. The plane of the loop makes an angle of 30° with the y−axis, and the current in the windings is 0.5 A.

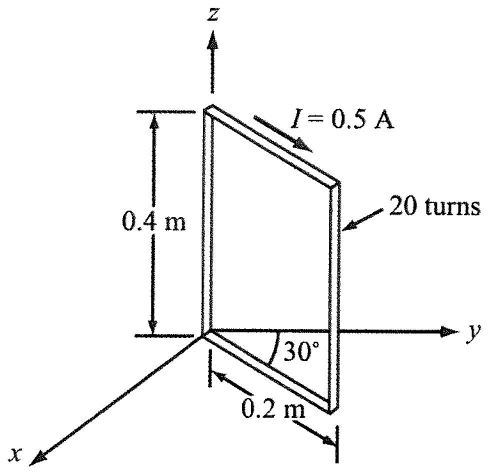

Figure 1: Hinged rectangular loop

(i) What is the magnitude of the torque exerted on the loop in the presence of a uniform field $\mathbf{B} = \hat{\mathbf{y}}2.4\ T$? [4]

(ii) When viewed from above, is the expected direction of rotation clockwise or counterclockwise? [2]

(b) Jean Biot and Felix Savart arrived at an expression that relates the magnetic field **H** at any point in space to the current I that generates **H**.

(i) Use infographics to illustrate the Biot-Savart law [3]

(ii) State the Biot-Savart law and write down its mathematical expression. [4]

(c) Two infinitely long, parallel wires are carrying 6 A currents in opposite directions. Determine the magnetic flux density at point P in Figure 2. [4]

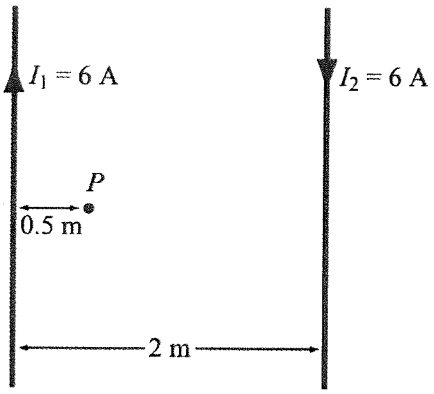

Figure 2: Arrangement for the two infinitely long, parallel wires.

(d) A circular loop of radius a carrying current I_1 is located in the $x-y$ plane as shown in Figure 3. In addition, an infinitely long wire carrying current I_2 in a direction parallel with the z-axis is located at $y = y_0$.

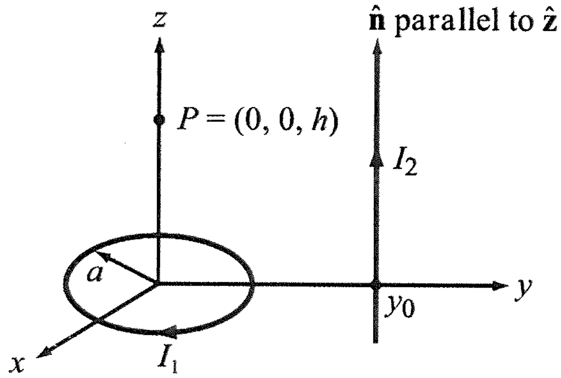

Figure 3: Arrangement for a circular loop and the infinitely long wire

(i) Determine the magnetic field $\mathbf{H}$ at point $P = (0, 0, h)$. [6]

(ii) Evaluate $\mathbf{H}$ for $a = 3$ cm, $y_0 = 10$ cm, $h = 4$ cm, $I_1 = 10$ A, and $I_2 = 20$ A. [2]

Part 4: Transmission-Line

Question 4 .... Total: 25

(a) (i) Under what set of circumstances should we explicitly treat the pair of wires as a transmission line? [3]

(ii) Sketch a lumped element circuit model representation of a transmission line. Label it clearly. [5]

(b) A two-wire copper transmission line is embedded in a dielectric material with $\epsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}\ S/m$. Its wires are separated by 3 cm and their radii are 1 mm each. Calculate the following line parameters at 2 GHz

(i) The combined resistance of both conductors per unit length R'. [4]

(ii) The combined inductance of both conductors per unit length L'. [2]

(iii) The conductance of the insulation medium between the two conductors per unit length G'. [2]

(iv) The capacitance of the two conductors per unit length C'. [2]

(c) A 50 Ω microstrip line uses a 0.5 mm thick sapphire substrate with $r = 9$. What is the width of its copper strip? [7]

Full marks: 100

Some useful vector identities

$A \cdot B = ABcos\theta_{AB}$, Scalar (or dot) product
$A \times B = \hat{n}ABsin\theta_{AB}$
$A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$
$A \times (B \times C) = B(A \cdot C) - C(A \times B)$
$\nabla \cdot (U + V) = \nabla U + \nabla V$
$\nabla \cdot (UV) = U \nabla V + V \nabla U$
$\nabla \cdot (A + B) = \nabla \cdot A + \nabla \cdot B$
$\nabla \cdot (UA) = U \nabla \cdot A + A \cdot \nabla U$
$\nabla \times (UA) = U \nabla \times A + \nabla U \times A$
$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A(\nabla \times B)$
$\nabla \cdot (\nabla \times A) = 0$
$\nabla \times \nabla V = 0$
$\nabla \cdot \nabla V = \nabla^2 V$
$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$
$\int_v (\nabla \cdot A) dV = \oint_S A \cdot ds$ Divergence theorm (S enclosed v)
$\int_S (\nabla \times A) \cdot ds = \oint_C A \cdot dl$ Stokes's theorem (S bounded by C)

GRADIENT, DIVERGENCE, CURLl, & LAPACIAN OPERATORS

Catersian (rectangular) coordinates (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times A$$

$$= \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix}$$

$$= \hat{x}(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}) + \hat{y}(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}) - \hat{z}(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinates (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot A = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times A$$

$$= \frac{1}{r}\begin{bmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & A_\phi & A_z \end{bmatrix}$$

$$= \hat{r}(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}) + \hat{\phi}(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}) - \hat{z}\frac{1}{r}(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi})$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial V}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinates (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{Rsin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot A = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{Rsin\theta}\frac{\partial}{\partial \theta}(A_\theta sin\theta) + \frac{1}{Rsin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times A$$

$$= \frac{1}{R^2 sin\theta}\begin{bmatrix} \hat{R} & \hat{\theta}_R & \hat{\phi}Rsin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (Rsin\theta)A\phi \end{bmatrix}$$

$$= \hat{R}(\frac{1}{Rsin\theta}[\frac{\partial}{\partial \theta}(A_\phi sin\theta) - \frac{\partial A_\theta}{\partial \phi}] + \hat{\theta}\frac{1}{R}[\frac{1}{sin\theta}\frac{\partial A_R}{\partial \theta} - \frac{\partial}{\partial R}(RA_\theta)] + \hat{\phi}\frac{1}{R}[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}]$$

$$\nabla^2 V = (\frac{1}{R^2}\frac{\partial}{\partial R}(R^2\frac{\partial V}{\partial R}) + \frac{1}{R^2 sin\theta}(\frac{\partial}{\partial \theta}(sin\theta\frac{\partial V}{\partial \theta}) + \frac{1}{R^2 sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Fundamentals physical constants

Constant	Symbol	Value
Speed of light in vacuum	c	$2.998 \times 10^8 \approx 3 \times 10^8 m/s$
Gravitational constant	G	$6.67 \times 10^{-11} N \cdot m^2/kg^2$
Boltzmann's constant	K	$1.38 \times 10^{-23} J/K$
Element charge	e	$1.60 \times 10^{-19} C$
Permittivity of free space	ε_0	$8.85 \times 10^{-12} \approx \frac{1}{36\pi} \times 10^{-7} F/m$
Permeability of free space	μ_0	$4\pi \times 10^{-7} H/m$
Electron mass	m_e	$9.11 \times 10^{-31} kg$
Proton mass	m_p	$1.67 \times 10^{-27} kg$
Plank's constant	h	$6.63 \times 10^{-34} J \cdot s$
Intrinsic impedance of free space	η_0	$376.7 \approx 120\pi\Omega$

Conductivity, $\sigma(S/m)$ of some common materials

Material	Conductivity,$\sigma(S/m)$	Material	Conductivity,$\sigma(S/m)$
Silver	6.2×10^7	Pure Germanium	2.2
Copper	5.8×10^7	Pure Silicon	4.4×10^{-4}
Gold	4.1×10^7	Wet soil	$\sim \times 10^{-2}$
Aluminium	3.5×10^7	Fresh water	$\sim \times 10^{-3}$
Tungsten	1.8×10^7	Distilled water	$\sim \times 10^{-4}$
Zinc	1.7×10^7	Dry soil	$\sim \times 10^{-4}$
Brass	1.5×10^7	Glass	$\times 10^{-15}$
Iron	10^7	Hard rubbers	$\times 10^{-15}$
Bronze	10^7	Paraffin	$\times 10^{-15}$
Tin	9×10^7	Mica	$\times 10^{-15}$
Lead	5×10^7	Fused quartz	$\times 10^{-15}$
Mercury	610^6	Wax	$\times 10^{-15}$
Carbon	3×10^4		
Sea water	4		
Animal body (Average)	0.3		

Dot products of unit vectors in cylindrical and rectangular coordinate systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1

Dot products of unit vectors in spherical and rectangular coordinate systems

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

Common Derivatives and Integrals

Exponential & Logarithm Functions

$$\int \mathbf{e}^u\,du = \mathbf{e}^u + c \qquad \int a^u\,du = \frac{a^u}{\ln(a)} + c \qquad \int \ln(u)\,du = u\ln(u) - u + c$$

$$\int \mathbf{e}^{au}\sin(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\Big(a\sin(bu) - b\cos(bu)\Big) + c \qquad \int u\mathbf{e}^u du = (u-1)\mathbf{e}^u + c$$

$$\int \mathbf{e}^{au}\cos(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\Big(a\cos(bu) + b\sin(bu)\Big) + c \qquad \int \frac{1}{u\ln(u)}\,du = \ln\left|\ln(u)\right| + c$$

Inverse Trig Functions

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + c \qquad \int \sin^{-1}(u)\,du = u\sin^{-1}(u) + \sqrt{1-u^2} + c$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c \qquad \int \tan^{-1}(u)\,du = u\tan^{-1}(u) - \frac{1}{2}\ln\left(1+u^2\right) + c$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + c \qquad \int \cos^{-1}(u)\,du = u\cos^{-1}(u) - \sqrt{1-u^2} + c$$

Hyperbolic Functions

$$\int \sinh(u)du = \cosh(u) + c \qquad \int \operatorname{sech}(u)\tanh(u)du = -\operatorname{sech}(u) + c \qquad \int \operatorname{sech}^2(u)du = \tanh(u) + c$$

$$\int \cosh(u)du = \sinh(u) + c \qquad \int \operatorname{csch}(u)\coth(u)du = -\operatorname{csch}(u) + c \qquad \int \operatorname{csch}^2(u)du = -\coth(u) + c$$

$$\int \tanh(u)du = \ln\left(\cosh(u)\right) + c \qquad \int \operatorname{sech}(u)du = \tan^{-1}\left|\sinh(u)\right| + c$$

Miscellaneous

$$\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + c \qquad \int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + c$$

$$\int \frac{1}{u^2-a^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + c \qquad \int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + c$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + c$$