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Faculty	Agriculture, Engineering and Natural Sciences
School	Science
Department	Computing, Mathematical and Statistical Science
Subject	Calculus I
Subject Code	MAT 3611
Date	June 2022
Duration	Three Hours
Marks	100

SUPPLEMENTARY EXAMINATION PAPER

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Moderator: Prof. J-B. Gatsinzi, BIUST

INSTRUCTIONS:

(i) This question paper consists of **FOUR** pages (***including*** this front page).

(ii) Answer **ALL** questions in section A and **ANY** 3 out of 4 questions in section B.

(iii) Only ***non-programmable calculators*** may be used.

(iv) Try to understand each question before you answer it.

(v) Number the questions clearly and present your solutions in a logical manner.

(vi) Use proper mathematical terminology.

(vii) The full marks for this paper is 100.

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EXAMINATIONS

Section A.[40 marks]

Answer **ALL** questions in this section.

Question A1.[14 marks]

A1.1 Give a precise definition of the following concepts.

a) $c \in \mathbb{R}$ is an accumulation point of a subset A of $\mathbb{R}$. [2]

b) $\lim_{x \to -\infty} f(x) = L$, where $L \in \mathbb{R}$. [3]

c) A point $(c, f(c))$ is a point of inflection for the graph of f. [4]

d) A function f is continuous from the left at $c \in D_f$. [2]

A1.2 State the first derivative test. [3]

Question A2.[16 marks]

Verify the following limits by applying the definition.

a) $\lim_{x \to 3} \left(\frac{x^3 - 1}{x^2 + 1} \right) = \frac{13}{5}$, for $\varepsilon = 0.01$. [10]

b) $\lim_{x \to 4^+} \left(\frac{2030}{\sqrt{x^2 - 16}} \right) = +\infty$. [6]

Question A3.[10 marks]

a) A wire of 100 cm is cut in two pieces to make a square and a circle. Find the length of each piece of wire so that the sum of the area enclosed by the square and the circle is minimum. [5]

b) Suppose f is a function that satisfies the equation

$$f(x + h) = f(x) + f(h) + x^2 h + x h^2$$

for all $x, h \in \mathbb{R}$. Suppose also that

$$\lim_{x \to 0} \frac{f(x)}{x} = 10.$$

(i) Find $f(0)$. [1]

(ii) Find $f'(0)$. [1]

(iii) Find the derivative function $f'(x)$ and give its domain. [3]

Section B. [60 marks]

Answer **ANY** 3 OUT OF 4 questions in this section.

Question B1. [20 marks]

a) Without using derivatives, valuate the following limit.

(i) $\lim\limits_{x\to 1}\left(\frac{|2x-3|-|2x-1|}{x-1}\right)$. [6]

(ii) $\lim\limits_{x\to 5^-}\frac{\sqrt{x-1}-2}{\sqrt[3]{2x-2}-2}$. [6]

b) Let $a, b \in \mathbb{R}$. Consider the function

$$f(x)=\begin{cases}\frac{\sqrt{a(x-2)+b}-4}{x-2} & \text{if} \quad x<2,\\ 2x-1 & \text{if} \quad x\geq 2.\end{cases}$$

Find the values of a and b, so that f is continuous at 2. [5]

c) Use the Intermediate Value Theorem to show that the equation $e^x = 4 - x^3$ has at least one solution. [3]

Question B2. [20 marks]

Consider the function

$$f(x)=\frac{\ln x}{x^3}$$

a) Find the domain of f. [1]

b) Find the x, y-intercepts. [2]

c) Find $\lim\limits_{x\to 0^+} f(x)$. Identify any possible asymptote. [3]

d) Find $\lim\limits_{x\to +\infty} f(x)$. Identify any possible asymptote. [3]

e) Find $f'(x)$ and $f''(x)$. [4]

f) Find the critical number(s) of f. [1]

g) Find the intervals of increase and decrease. [2]

h) Discuss the concavity of f and give any possible point(s) of inflection. [2]

i) Sketch a well labelled graph of f. [2]

Question B3. [20 marks]

a) Use implicit differentiation to find y' , where $2^{-x}y^2 - x + \log_2(yx^3) = \tan^{-1}(\ln x)$. [3]

b) Evaluate the following integral.

(i) $\int_2^3 \frac{e^x}{e^{2x}-1}dx$. [8]

(ii) $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$. [4]

c) Scientists can determine the age of ancient objects by radiocarbon dating. The half-life of radioactive carbon $14C$ is 5730 years. A discovery of a parchment revealed that it had 74% as much $14C$ radioactivity as plant material does today. What is the age of the parchment? [5]

Question B4. [20 marks]

a) If a and b are distinct positive real numbers, find the maximum value of [8]

$$f(x) = (x-1)^b(2-x)^{2a}, \text{ where } 1 \leq x \leq 2.$$

b) Determine the number(s) $a \in \mathbb{R}$ for which the function f has no critical number: [8]

$$f(x) = (a-6)\cos(2x) + (a-2)x + \sin(2022)$$

c) If f is twice differentiable at c, where $c > 0$, evaluate the following limit in terms of $f''(c)$:

$$\lim_{x \to c}\left(\frac{f'(x) - f'(c)}{\sqrt{x} - \sqrt{c}}\right)$$

[4]

END